Chapter 1

The Big Game and Radioactive Decay: INTRODUCTION TO MODELS

IM-1. e) The equation depends on the tubes used. The students should have a slope between 0.12 and 0.14 with a y-intercept around 1.5 inches if they use a paper towel core.

IM-2. a) 6 - 7 yards b) 20 - 24 yards

IM-3. a) It is a trapezoid with bases centered at opposite ends of the field
b) 1560 - 1860 square yards c) 24% - 29%

IM-4. 37% - 44%

IM-5. a) parabola b) cubic
c) hyperbola or inverse variation d) exponential
e) absolute value f) square root

IM-6. The most likely answers are a circle and $x = y^2$. Students may draw different examples of non-functions. You may wish to ask for student responses to this question.

IM-7. a) 114.602 b) 10.705

IM-8. a) $y = \frac{9}{4}x \pm 1$ b) $y = \frac{9}{4}x + 47$

IM-9. a) 29 b) 47 c) $18b^2 - 3$ d) $2a^2 + 4a - 1$

IM-10. a) $x + 2$ b) $2x - 1$ c) $x^2 + 4$ d) $5x$

IM-11. c) They are half as large each time. Divided by 2 or multiplied by $\frac{1}{2}$ would also be acceptable.

IM-12. a) $2^{\ 2}$ b) $2^{\ 3}$ c) $2^{\ 3}$

IM-13. a) $x(2x + 5)$ b) $3xy^3(xy^2 - 3)$ c) $17x^2y(xy - 2)$

IM-14. a) 15 b) $(9, \frac{15}{2})$

IM-15. a) $8x^3$ b) $9x^2 - 12x + 4$ c) $81x^4$ d) $\frac{1}{27x^3}$

IM-16. a) (7, 2) b) $\frac{1}{2}$ c) (x, 2) d) $\frac{y\ \ 2}{x\ \ 1}$ e) $y \pm 2 = \frac{1}{2}(x \pm 1)$
f) (x, y_1) g) $y - y_1,\ x - x_1,\ \frac{y\ \ y_1}{x\ \ x_1}$

IM-18. a) $y + 3 = \frac{3}{5}(x \pm 10)$ b) $y \pm 7.3 = 2.85(x \pm 6.1)$
c) $y \pm 8 = \frac{13}{11}(x \pm 4)$ or $y \pm 21 = \frac{13}{11}(x \pm 15)$
d) $y - 6.24 = 4.425(x - 4.3)$ or $y - 9.78 = 4.425(x - 5.1)$

IM-20. c) $0 < b < 1$

IM-21. a) $y = \frac{3}{5}x \pm 9$ b) $y = 2.85x - 10.085$

IM-22. slope $= -\frac{5}{2}$

IM-23. opposite reciprocal

IM-24. when slope is zero

IM-25. a) $y - 7 = 3(x + 2)$ b) $y - 20 = -\frac{4}{3}(x - 12)$ c) $y - 3 = \frac{1}{2}(x - 5)$

IM-26. $9x^2 + 12x + 4$

IM-27. a) yes

IM-28. a) $a^{(b+c)}$ b) $a^{(c-b)}$
c) This is impossible; we are not multiplying d) $a^{(b+1)}$ e) a^b
f) $a^{(3c+b)}$

IM-29. a) $3x^2y(27x - 4)$ b) $(2x + 1)(x + 8)$ c) $(3x - 7)(7x - 16)$
d) $(x + y)(m + x + y)$

IM-30. a) $25a^{-4}$ b) $m^{-3}n^{-6}$ c) $8x^{-2}$

IM-31. a) 2^{2x} b) 2^{-20} c) $\frac{3}{2}$ d) $\frac{9}{4}$

IM-34. shifted up 5

IM-35. $y = x^2 - 4$

IM-36. shifted left 2

IM-37. $y = \sqrt{x \pm 3}$

IM-38. parent: $y = \frac{1}{x}$, transformation: left 4 and down 3

IM-39. a) 6 miles d) $\frac{\textbf{miles}}{\textbf{hr}} \cdot \text{hr} = \text{miles}$

IM-40. b) Set 1: mean = 8.8, median = 10; Set 2: mean = 14.8, median = 10
c) the mean d) High-priced homes will have too much of an influence on the mean. If a neighborhood had homes around $100,000 and a few homes worth over a million dollars, the mean may not be a good estimate of the typical price that would be paid for a house in the area.

IM-41. a) $y = \frac{1}{x} - 2$ b) $y = (x + 2)^3 + 3$

IM-42. a) 150 miles b) It is a rectangle
c) height = 50 mph, base = 3 hours, $\left(\frac{50 \text{ miles}}{\text{hr}}\right)(3 \text{ hr}) = 150 \text{ miles}$

IM-43. a) 3^{15} b) $2^{(-3x)}$ c) $2^{(9x-10)}$

IM-44. a) $x = -\frac{\textbf{4}}{\textbf{3}}$ b) $x = \frac{\textbf{1}}{\textbf{4}}$ c) $x = \frac{\textbf{11}}{\textbf{6}}$

IM-45. a) It multiplies the input by two and then adds 1
b) We hope that they will think that 3 would come out the top and if that's true, then the machine must "undo" itself—work backwards in a sense
c) Subtract one and then divide by two

IM-46. a) Subtract 6, then multiply by 2 b) $f^{-1}(x) = 2(x - 6)$

IM-47. a) $x^2 + 3x - 3$ b) $3x^3 - 5x^2 + 6x - 10$ c) $3x^2 + 1$
d) $9x^2 - 30x + 27$

IM-48. a) $3x^3 - x^2 + 8$ b) $\frac{3x^3+7}{x^2\ \ 1}$ c) $3x^5 - 3x^3 + 7x^2 - 7$

IM-49. a) -37 b) -5 and 2 c) 4 and 3 d) 0, 2, and 3

IM-50. b) We want the sections to be "balanced"—right and left strips should have the same number of points
c) (2, 10), (9, 5), and (16, 3) d) $y = -0.5x + 11$
e) 6.5, 1.5 units above the point
f) Shift the graph down 0.5 units; final equation $y = -0.5x + 10.5$

IM-52. a) 112.5 miles b) two rectangles c) 112.5 miles
d) miles = mph · hours

IM-53. a) $\sqrt{x \pm 1}$ b) 15 c) 9 d) 15 e) $x^2 + 4x + 3$
f) $|x + 1|$

IM-54. $a + b = 1$

IM-55. a) -5^4 , 5^{-4} or $\frac{1}{5^4}$ b) -3^{-5} , 3^5 c) 11^{-6} , -11^6
d) $-\frac{2}{7}$, $\frac{7}{2}$ e) $-\left(\frac{11}{9}\right)^2$, $\left(\frac{9}{11}\right)^2$ f) $\left(\frac{7}{13}\right)^5$, $\left(\frac{7}{13}\right)^5$

IM-57. a) $2xy^3(2x - 3y)$ b) $2(x + y)(x - y)$ c) $(x - 3)(x - 2)$

IM-58. $x = 2,\ y = 3$

IM-59. a) $y - 4 = \frac{2}{3}(x + 1)$ b) $y - 4 = -\frac{3}{2}(x + 1)$

IM-60. a) $\pm\ \frac{1}{8}$ b) $\pm\sqrt{\frac{1}{8}}$ or $\pm\frac{\sqrt{2}}{4}$

IM-61. a) $9x^2 + 16x + 3$ b) $-9x^3 - 27x^2 + 24x$

IM-62. a) If you scale the horizontal axis by 5 and the vertical by 25, all of the data will fit nicely b) $y = 8.864x - 506$ c) approx. 274 grams
d) 38.3% error e) no; besides the error analysis from part (d), students should observe that the data are not linear and for smaller values of x, the model will give a negative net weight.

IM-63. a) It is vertically stretched b) It is vertically stretched.

IM-64. a) $y = x^3$ b) $y = \frac{1}{2}x^3$ c) It is compressed vertically.

IM-65. Use a negative vertical stretch factor.

IM-66. $x = -5$

IM-67. a) $p = 6,\ q = 2$ b) not in form p^q c) $p = x + 3y;\ q = 2 - r$
d) not in form p^q

IM-68. a) 6^{-4} b) 5^7 c) $\frac{3}{8}\Diamond 19^3$ d) $\frac{3}{8}\Diamond 19^{\ 3}$

IM-70. a) impossible b) impossible c) 2^5 d) 2^6 e) 2^{-1}
f) impossible

IM-71. $\frac{2(x+3)}{5}$

IM-72. a) $x(x + 8)$ b) $6x(x + 8)$

IM-73. $x = 5$

IM-74. a) 2^8 b) $\frac{64}{25}$

IM-75. a) $a(b + c – d)$ b) $(2y – 3)(x + 2)$

IM-76. a) It is a vertical stretch c) (0, 3)
d) It is the same as the vertical stretch factor.

IM-77. a) A vertical stretch of 3 and shift up of 1 c) $y = 1$ d) (0, 4)
e) It is the stretch factor (3 units) above the asymptote f) (0, A + k)

IM-78. a) $f(x) = 8 \cdot 2^x$ b) $f(x) = 6 \cdot 2^x + 4$
c) $f(x) = 2 \cdot 2^x – 2$ d) $f(x) = -2 \cdot 2^x + 2$

IM-79. a) It is shifted left 2 units and stretched b) (0, 12) c) $g(x) = 12 \cdot 2^x$
d) The students may state that since the graphs are the same, the two functions are equal. The goal is to have them use the rules of exponents to convert f(x) to g(x). This process will be done in the next problem, which will help the students who do not see it here.

IM-81. a) 45 b) 25 c) 256 d) $\frac{1}{81}$

IM-82. a) $18 = a \cdot b^2$, $162 = a \cdot b^4$ b) $a = \frac{\mathbf{18}}{\mathbf{b^2}}$, $162 = 18b^2$ c) $b = 3$
d) $a = 2$ e) $y = 2 \cdot 3^x$

IM-83. a) $\mathbf{\frac{20}{3}}$ b) $\mathbf{\frac{81}{2}}$ c) $\mathbf{\frac{1}{8}}$ d) 49

IM-84. $x = -4$ or -2

IM-85. $\sqrt[6]{90}$

IM-86. $y = \sqrt[n]{x}$

IM-87. a) 16 b) 625 c) 27 d) $\frac{4}{9}$

IM-88. $y = -\frac{1}{2}x + \frac{15}{4}$

IM-89. a) 5^{-1} b) 3^7 c) $\mathbf{\frac{5}{8}} \cdot 14^3$ d) $\mathbf{\frac{5}{8}} \cdot 14^{-3}$

IM-90. a) $b = 2$ b) $A = 0.75$

IM-91. a) 4 b) 1000 c) 25

IM-92. $\frac{3}{7}$

IM-93. 9 g, 4.5 g, 2.25 g, 1.125 g, 0.562 g

IM-94. A = 100, b = 0.5

IM-95. 4, 5, 6; 9 g, 4.5 g, 2.25 g, 1.125 g, 0.562 g

IM-96. c) A = 36, b = 0.794

IM-98. a) 24.3 g, 21.87 g, 19.683 g, 17.715 g, 15.943 g b) each ratio = 0.9
c) both = 0.9 e) On calculators that only use two digits for the exponent part of scientific notation, the second fraction in part (c) will give a result of zero.

IM-99. a) about 5700 years b) about 11,400 years c) about 17,100 years
d) about 22,800

IM-100. $C(t) = A(0.5)^{t/5700}$ or $C(t) = A(0.999878)^t$

IM-102. a) $x^3 - 6x^2 - 16x$ b) $2x^2 + \frac{7}{2}x + 4$ c) $x^3 - 6x^2 - 16x$
d) $x^2 + 2x - 4G$

IM-103. $A = 0.5^A$ so A = 0.641

IM-104. 2, 3, 4, 5; 2, 4, 6, 8 a) The values are interchanged
b) x = 2y – 2; it shows that f and f^{-1} are inverses of each other.

IM-105. a) $f^{-1}(x) = \frac{x \pm 4}{2}$ b) $g^{\pm 1}(x) = \sqrt[3]{x \pm 1}$

IM-106. They are reflections of each other across the line y = x.

IM-107. a) $f^{-1}(x) = \frac{x \pm k}{m}$ b) $f^{-1}(x) = \sqrt[3]{x \quad 5}$

IM-108. 20 mph

IM-109. a) 10 days b) $I(t) = 100 \cdot (0.5)^{t/10}$ c) 0.000 d) The amount left is less than $\frac{1}{10000}$ mR. The value in part (c) is $7.88 \cdot 10^{-29}$.

IM-110. a) 25 b) 25 c) 25
d) The ratios have terms separated by two units, giving a result of $5^2 = 25$.

IM-111. a) b b) b^2 c) b d) b^2

IM-112. a) $b = \sqrt{10} = 3.162$ b) $A = \frac{2}{5\sqrt{10}}$ or $\frac{\sqrt{10}}{25} = 0.126$

IM-113. a) $b = \sqrt{10} = 3.162$ b) $A = \frac{1}{5\sqrt{10}}$ or $\frac{\sqrt{10}}{50} = 0.063$

IM-114. a) 18.75 g b) $150(0.5)^{t/57}$ c) 223 hours

IM-115. The sample is 68.076% Br^{76} and 31.924% Br^{77}.

IM-116. $y = 0.8x + 2.4$

IM-117. $6\frac{1}{9}$

IM-118. a) $y = \frac{4}{3}x \quad 12$ b) $y = x + \frac{1}{6}$, $x \neq -\frac{1}{6}$

IM-119. a) $(2x - 3y)(2x + 3y)$ b) $2x^3(2 + x^2)(2 - x^2)$

IM-120. $x = \pm\sqrt{7 \pm b}$, $b \quad 7$

IM-121. a) 9 b) 8

IM-122. $y - 12 = -\frac{19}{2}(x \pm 3)$, $\sqrt{365}$

IM-123. 12 gallons

IM-124. a) $\sqrt{68}$ b) $y = -4x - 14$, $y - 2 = -4(x + 4)$, $y + 6 = -4(x + 2)$

IM-125. $x < 3$

IM-126. a) the coefficients a, b and c b) $\frac{b + \sqrt{D}}{2a} = R$ and $\frac{b \pm \sqrt{D}}{2a} = S$
c) R and S; the Quadratic Formula has two solutions because of the ± in the formula

IM-128. a) $x = y^2$ c) $y = |x|$, $y = x^4$, $y = x^3 - 4x$ for example

IM-129. a) $y = \sqrt{x}$ b) $y = \sqrt{x} + 2$

IM-130. a) 54 b) $\frac{108}{\sqrt{180}} = \frac{18}{\sqrt{5}}$ or $\frac{18\sqrt{5}}{5}$

IM-131. $x = 2$, $y = 1$

IM-132. a) $2(x + 2)(x + 2)$ b) $6(x + 3)(x – 4)$

IM-133. $x = \frac{3}{11}$

IM-134. a) x 3 b) shaded under the line $y = -2x + 8$

IM-135. a) 29.74˚ b) 60.26˚

IM-136. 21 and 28 centimeters

IM-137. $f^{-1}(x) = \pm\sqrt{\frac{x-4}{2}}$

IM-138. a) $(a – 3)(a – 4)$ b) $(x – 3)(5x + 4)$

IM-139. a and e, b and c

IM-141. a) $f(x) = 7.8125(0.8)^x$ b) 0.839 and 1.59×10^{-9}

IM-142. a) $\frac{(x \pm 1)^3}{x+1}$ b) $\frac{4a^3}{a+1}$ c) $\frac{4}{a^3 + a^2} = \frac{4}{a^2(1+a)}$

IM-143. a) $A(t) = 20\left(\frac{3}{5}\right)^{t/8}$ or $20(0.93814...)^t$ b) 4.320 mg

IM-146. $y = \frac{1}{2}x + \frac{4}{3}$

IM-147. $a = 25, b = 5$

IM-148. a) $\sqrt{208} = 4\sqrt{13}$ b) (0, 1) c) $y – 7 = -\frac{3}{2}(x+4)$, $y + 5 = -\frac{3}{2}(x \pm 4)$

IM-149. a) $x^3 – 2$
c) 6, 2; composing f and f^{-1} in either order returns the original number

IM-150. $(32 + 4\ )m^2$

IM-151. $a = d$, $b = -2de$, and $c = de^2 + f$

IM-153. a) 15 b) 1 c) 222d) 7 e) $a^2 + 6a + 6$
f) $-a^2 + 2a + 8$ g) They are not equal.

IM-154. 187

IM-155. a) mean = 66.57, median = 67 b) mean = 69.86, median = 67
c) The median is not influenced by the high score, but the mean is.

IM-156. $x^2 + y^2 = 49$

IM-157. $(x + 2)(x + 10)$

