Chapter 3

The Spring Problem: SINUSOIDAL FUNCTIONS

SP-1. c) They should be d) 2 radius lengths = circumference

SP-3. 1 Unit

SP-5. C = 2

SP-6. a) 360˚ b) 2 radians c) 6

SP-8. a) 180˚, b) radians = 180˚ c) 60˚ d) $\frac{10}{9}$

SP-9. a) 57.296˚ b) 0.017 c) Very different; a radian is much larger.

SP-10. a) b) $-\frac{}{5}$ c) $\frac{2}{90}$

SP-11. a) 270˚ b) -210˚ c) $\frac{360}{}$˚

SP-12. a) 1 b) s^2 c) c^2 d) no unique value e) 4c f) $4c^2$

SP-13. a) x = 5.25 b) x = $\frac{28}{3}$

SP-14. $\frac{50}{3}$

SP-15. 0, $\frac{}{12}$, $\frac{}{6}$, $\frac{}{4}$, $\frac{}{3}$, $\frac{5}{12}$, $\frac{}{2}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{11}{12}$,

SP-16. a) 81 b) 648 c) $\frac{1}{\sqrt{5}}$

SP-17. a) $2\sqrt{3}$ b) $\frac{\sqrt{2}}{2}$

SP-18. a) $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$ b) $\frac{1}{2}, \frac{\sqrt{3}}{2}$

SP-23. a) $\frac{}{6}$ b) $\frac{5}{6}$ c) $\frac{7}{6}$ d) $\frac{11}{6}$, $\frac{}{6}$ e) $\frac{13}{6}$, $-\frac{23}{6}$

SP-25. a) (-0.6, 0.8) b) (0.9, -0.5)
c) (-0.2, -0.9) d) same as (a)

SP-26. a) 2 b) -1 c) 4 d) impossible e) 25 f) $\sqrt{5}$

SP-27. $x = \frac{9}{1^{3}}$

SP-28. a) $3\sqrt{2} + 7\sqrt{7}$ b) $6\sqrt{2}\quad 3\sqrt[3]{2}$ c) $5\sqrt{5}$ d) $9\sqrt{3}$

SP-29. a) 245 miles c) 245

SP-30. $h = \frac{12x}{x+4}$

SP-31. a) $\frac{4}{15}$ b) $-\frac{25}{9}$ c) 195° d) -400°

SP-32. a) $x = \frac{\sqrt{3}}{2}$, $y = \frac{1}{2}$ b) $x = \cos 40°$, $y = \sin 40°$
c) $x = \cos$, $y = \sin$

SP-33. a) (–, +) b) (–, –) c) (+, +) d) (–, 0)

SP-34. $x = \cos$, $y = \sin$

SP-35. a) $\tan = \frac{y}{x}$ b) i) I, II, ii) I, IV, iii) I, III

SP-37. a) (-0.6, 0.8) b) -0.6 c) 0.8

SP-38. a) $\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{2}}{2}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$

SP-39. b) -0.4161468365... c) 0 d) 1 e) 0
f) not defined

SP-40. a) -0.6 b) -0.392 c) Impossible; there is no such point.

SP-41. a) $-\frac{\sqrt{3}}{2}$ b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) $-\frac{1}{2}$

SP-42. a) $ab^{2}\sqrt{b}$ b) $2\sqrt[3]{3}$ c) $xy^{2}\sqrt[3]{x^{2}}$ d) $xy\sqrt[5]{y}$

SP-43. $x = \frac{7+b}{a+3}$, $y = \frac{21\ ab}{a+3}$

SP-44. a) Interpreted the negative exponent as opposite instead of reciprocal
b) Confused -2 exponent with $\frac{1}{2}$, so took root instead of reciprocal
c) Multiplied base by exponent instead of raising to a power.

SP-45. a) 2 b) 4 c) Domain: all numbers, Range: -2 y 2
d) odd numbers; r = 2n + 1

SP-46. a) 286.5˚ c) 20.944 meters
d) No—it falls just short of the shaded region.

SP-47. b) $\frac{11}{6}$, $\frac{7}{4}$, $\frac{5}{3}$

SP-48. a) D: -• x • , R: -1 y 1 b) D: -• x • , R: -1 y 1

SP-49. a) 2 , 2 b) 2 c) $\frac{5}{2}$

SP-51. a) We get more cycles when the number increases
b) Number of cycles in 2
c) It tells you how frequently the curve completes its cycles
d) , $\frac{2}{3}$, 4 e) pb = 2

SP-52. -0.8

SP-53. a) 0.36 b) 0.36 c) impossible d) -0.8
e) 1.92 f) 5.76

SP-54. a) $x^2 + y^2 = 1$ b) x = cos c) y = sin
d) $\cos^2 + \sin^2 = 1$ e) $\tan = \frac{\sin}{\cos}$

SP-56. b) $\pm\frac{4}{5}$ c) $\frac{\pm 4}{3}$

SP-57. a) $-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}$ b) $\frac{\sqrt{15}}{4}$
c) $\frac{\sqrt{15}}{4}, -\frac{\sqrt{15}}{4}, -\frac{\sqrt{15}}{4}$ d) $\boxed{\sqrt{15}}, \boxed{-\sqrt{15}}, \boxed{\sqrt{15}}, \boxed{-\sqrt{15}}$

SP-58. a) 1 b) $\sin^2$ c) $\sin^2 5$]
d) $\cos^2$ e) $\cos^2(3\ )$ f) $4\cos^2(3\ )$

SP-59. $\frac{4x^2}{25}$

SP-60. a) 20 g b) 270 g

SP-61. a) $x = \log_7 y$ b) $x = \sqrt[7]{y}$ c) $x = 5^y$

SP-62. $-\frac{12}{13}, -\frac{5}{12}$

SP-63. a) $-\frac{\sqrt{2}}{2}$ b) $-\frac{1}{2}$ c) $-\frac{\sqrt{3}}{2}$ d) 1

SP-64. c) $y = \sin(x - \frac{}{2}) + 3$ d) $y = \sin(x - h) + k$

SP-65. a) They're not. The only difference is a shift.
b) Shift right $\frac{}{2}$, but other answers are possible.
c) $y = \cos(x - \frac{}{2})$ d) $y = \sin(x + \frac{}{2})$
e) $\sin x = \cos(x - \frac{}{2})$, $\cos x = \sin(x + \frac{}{2})$

SP-66. a) $y = 0.5 \sin x$ b) $y = -\sin x$ c) $y = a \sin x + k$

SP-67. $y = a \sin(x - h) + k$

SP-68. a) Same amplitude. $y = \sin(2x)$ has half the period.
b) $p = \frac{2}{b}$, $b = \frac{2}{p}$, or $bp = 2$

SP-69. 1) a 2) a 3) f 4) b
5) c 6) g 7) h 8) e

SP-71. a) They are the same. b) $y = -\cos x$
c) Moves angle 2 quadrants ahead.

SP-72. a) A radian is an angle measure found by wrapping a radius around a circle.
b) $\frac{4}{3}$

SP-73. a) -0.62 b) -0.785 c) IV d) II

SP-74. $x = \frac{3c \quad 7}{a^2+b^2}$

SP-75. 7.5 to 8 is reasonable.

SP-76. a) Any angle in the 4th quadrant. b) Any angle in the 3rd quadrant.

SP-77. $b = \frac{3}{2}x$

SP-78. a) $\frac{t^2+1}{t^2}$ b) $\frac{x}{x^2+y^2}$ c) $x^2 + y^2$ d) $a^2 - b^2$

SP-79. a) $\sqrt{3}$ b) $\sqrt{2}$ c) 3

SP-80. a) $\frac{c}{a}$ b) $\frac{c}{a}$ c) $\frac{c}{b}$

SP-81. a) $\frac{y}{x}$ b) $\frac{1}{x}$ c) $\frac{1}{y}$ d) $\frac{x}{y}$

SP-82. a) 7.086 b) 3.404 c) -1.236

SP-83. $\tan = \frac{\sin}{\cos}$, $\cot = \frac{\cos}{\sin}$

SP-84. a) sin b) csc c) tan d) $\cot^2$

SP-85. a) $\tan^2 + 1 = \sec^2$ b) $1 + \cot^2 = \csc^2$

SP-87. a) 2 b) $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$ c) $\sqrt{3}$ d) $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

SP-88. a) $-\frac{\sqrt{3}}{2}$ b) -1 c) $\frac{\sqrt{2}}{2}$ d) $\frac{\sqrt{3}}{2}$

SP-89. a) Cut period in half to , change amplitude to 2, turn upside down, and raise 3 units
b) Shift units to the right, change amplitude to 3, and shift down 1 unit.

SP-90. a) $-\frac{\sqrt{3}}{2}$ b) $3\sqrt{6}$ 2 c) 5 $2\sqrt{6}$

SP-91. $3\sqrt{5}$

SP-94. a) Where sin x = 0, csc x will have vertical asymptotes.
c) $0 < \sin x < 1$; $1 < \csc x <$ •

SP-96. Secant: Domain: x π n , Range: y -1 or $y \geq 1$
Cosecant: Domain: x π $\frac{}{2}$ + n , Range: y -1 or $y \geq 1$

SP-97. a) $\frac{\sin A}{1 \sin^2 A}$ b) $\frac{\sin^3 A+1}{\sin A}$
c) $\frac{2}{\sin A}$ d) $\frac{2}{\sin^2 A}$

SP-98. a) $\frac{3}{2}$ b) $-\frac{\sqrt{5}}{3}$ c) $-\frac{2\sqrt{5}}{5}$ d) $-\frac{\sqrt{5}}{2}$

SP-99. a) Amplitude = 2, Shift down 1
b) $y = 2\sin(x + \frac{}{2})$ 1 is one possibility.

SP-101. a) $-\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1

SP-102. a) $\cos^2 A$ b) $\frac{1}{\cos A}$ c) $\frac{2}{\cos A}$ d) $\frac{2\cos^2 A \pm 1}{\cos^4 A}$

SP-103. $h = \frac{10x}{x+5}$

SP-104. a) $y = 3\cos 2x$ b) $y = -2\sin x + 1$
c) $y = \csc x$ d) $y = 2\cos(0.5x) - 1$

SP-105. 1500 feet

SP-106. 2.5

SP-107. a) $x = 3$ or -3
c) It shows that the two graphs intersect more than once.

SP-108. $\frac{}{6}, \frac{5}{6}$

SP-109. a) Since the line graphs intersect at more than one point, there must be more than one solution to the equation.
b) There are 2 solutions: $x = \frac{}{6}, \frac{5}{6}$.
c) It shows where the y-coordinate = 0.5.
d) There are 2 solutions: $x = \frac{4}{3}, \frac{5}{3}$

SP-110. Draw a vertical line at $x = \frac{1}{2}$; $x = \frac{}{3}, \frac{5}{3}$

SP-111. A horizontal line drawn at $y = 2$ does not intersect the unit circle. The value 2 is not in the range for $y = \sin x$.

SP-112. a) $\frac{3}{2}$ b) $\boxed{\frac{2}{3}, \frac{4}{3}}$ c) $\boxed{\frac{3}{4}, \frac{5}{4}}$ d) $\boxed{\frac{}{3}, \frac{2}{3}}$

SP-113. tangent

SP-114. a) Since the string is 30 inches in length, the maximum point will be 30 inches above the minimum.

b) 15 c) 20 d) $\boxed{\frac{4}{5}}$

e) negative cosine f) $h = -15\cos\left(\frac{4}{5}(t)\right) + 20$

SP-115. a) 2 b) undefined c) $-\frac{\sqrt{3}}{3}$ d) $-\frac{2\sqrt{3}}{3}$

SP-116. a) 0, 2 b) $\frac{3}{4}, \frac{7}{4}$ c) $\frac{}{6}, \frac{5}{6}$ d) $\frac{}{4}, \frac{7}{4}$

SP-117. a) $10\sqrt{2}$ $8\sqrt{3}$ b) $2\sqrt{3}$ c) $\frac{2\sqrt{6}}{3}$

SP-118. $h = 5\cos\left(\frac{}{2}(t)\right) + 3$ SP-119. $\sqrt{5}$ SP-120. $A = 12,\ B = \frac{4}{3}$

SP-121. Amp = 8, vertical shift = 12, period = 8, angular frequency =

$\frac{}{4}$

SP-122. Period = $\frac{1}{5}$ second. Frequency = 10p

SP-123. b) Since cosine starts at a peak,we will not have to incorporate a horizontal shift.

SP-124. The period stays consistent regardless of the oscillations. SP-125. $\frac{\text{period}}{2}$

SP-127. a) We are starting at a peak, which is the same for cosine. b) k

SP-129. No—the height of the oscillations will decrease with time.

SP-130. Only the amplitude is affected. We observed earlier that the period stays consistent. The slinky will oscillate up and down until it comes to rest in the middle position.

SP-131. The graph is approaching the vertical shift.

SP-132. a) $\boxed{\frac{}{2}, \frac{3}{2}}$ b) $\frac{3}{4}, \frac{5}{4}$
c) same as (a) d) same as (b)

SP-133. a) 0.412, 2.730 b) 1.245, 5.038
c) 1.107, 4.249 d) 2.419, 3.864

SP-134. I = 0.8 sin (120 t)

SP-135. a) $x^{16}y^8$ b) $24x^5y^7$ c) $xy^3\sqrt{x}$ d) $\frac{3}{2x^5y^5}$

SP-136. a) Moved $\frac{}{2}$ units left b) Up 20, amplitude = 3

SP-137. 13.8 grams

SP-138. $h = \frac{1}{x}$

SP-139. a) k b) a

SP-141. a) y – 26.746 = -.193(x – 1) or y = 26.939 - .193 x
b) y = 25.202 and y = 25.009

SP-142. b) m = .95778 and a = 4.772 c) y = 25.412 and 25.275

SP-143. Exponential decay is better

SP-144. a) tan = $\frac{\text{opp}}{\text{adj}} = \frac{\text{BC}}{1}$

b) APB and are complementary, APB and A are complementary; therefore A = .

c) AA similarity

d) $\frac{\text{AP}}{\text{BP}}$

e) substituting the known lengths in the proportion in part (d) we get $\frac{1}{\sin} = \frac{\text{AP}}{1}$

f) $\frac{\text{PB}}{\text{PD}} = \frac{\text{PC}}{\text{PB}}$; therefore, $\frac{1}{\cos} = \frac{\text{PC}}{1}$

g) ABP ~ PDB ~ PBC, $\frac{\text{PB}}{\text{BC}} = \frac{\text{AB}}{\text{PB}}$; therefore, $\frac{1}{\tan} = \frac{\text{AB}}{1}$.

SP-145. a) $\sin^2 + \cos^2 = 1$ b) $\tan^2 + 1 = \sec^2$ c) $1 + \cot^2 = \csc^2$

SP-146. Pythagorean Theorem applied to PAC.

SP-147. $\frac{4}{25}(10 \quad h)^2$

SP-148. a) $y = -3 \sin x + 1$ b) $y = 2 \cos 3x - 1$
c) $y = \sec x$ d) $y = 2 \sin (x - \frac{}{4}) + 1$

SP-148. Find reasonable equations for each of the following graphs:

SP-149. c) $h(x) = \begin{cases} (x \quad 3)^3 + 4, & 2 \quad x \quad 4 \\ 5, & x > 4 \end{cases}$

SP-150. a) 5.96 mph
b) Spent much more time at 3 mph

SP-151. a) 120 b) 720 c) 3,628,800

SP-152. tan x

SP-153. $-\frac{4}{3}$

SP-154. a) The exponential function approaches the resting position of the spring.
b) 1.7 seconds c) $p = \frac{x}{1.7}$ d) $4.772(.95778)^{(x/1.7)}$

SP-156. a) D: $(-\bullet, \bullet)$; R: $[1, \bullet)$ b) D: $[5, \bullet)$; R: $[-2, \bullet)$
c) $(-\bullet, \bullet)$; R: $(-4, \bullet)$ d) $(3, \bullet)$; R: $(\bullet, \bullet)$

SP-157. a) $x = \frac{3}{7}$ b) $x = \frac{1}{63}$ c) $x = 2$ d) $x = 6$ and $x = 3$

SP-158. $g(x) = \begin{cases} (x+2)^2 & \text{for } x < 2 \\ x & \text{for } x \geq 2 \end{cases}$

SP-159. $x = \frac{24}{5}$

SP-161. 5

SP-162. $y = 20 \cos(\frac{}{2}x) + 30$

SP-163. $x = \frac{}{3}, \frac{5}{3}$

SP-165. Physical = $100 \sin(\frac{2}{23}(x))$, Emotional = $100 \sin(\frac{2}{28}(x))$, and Intellectual = $100 \sin(\frac{2}{33}(x))$

SP-167. Her age on this date is 5982 days. Her intellectual peak is at about 99.

SP-168. $\cos = \frac{\sqrt{5}}{3}, \tan = \frac{2\sqrt{5}}{5}, \csc = \frac{3}{2}, \sec = \frac{3\sqrt{5}}{5}, \cot = \frac{\sqrt{5}}{2}$

SP-169. a) $\frac{7}{6}, \frac{11}{6}$ b) 1.266, 5.017 c) $\frac{}{3}, \frac{4}{3}$ d) $\frac{}{4}, \frac{3}{4}$

SP-170. a) sec A b) 1

SP-171. a) $y = 2 \sin(\quad \frac{}{2}) + 2$ b) $y = -2 \cos \quad + 2$

SP-172. a) $-\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{2}}{2}$ c) $-\sqrt{3}$ d) undefined

SP-173. -1.25

SP-174. $d = \frac{14+t}{\tan}$

SP-175. a) 1.5 $r + 2r \quad 6.712r$ b) $\frac{4}{3} \quad r + 2r\sqrt{3} \quad 7.653r$

SP-176. $x = 0, -\frac{5}{2}$

SP-177. $x = -1$

