Chapter 4

Big Ideas and Little Tricks: ALGEBRA FOR COLLEGE MATHEMATICS COURSES

AL-1. a) $u = \sqrt{x^2 + 2}$ b) $u = 4 \ \ -2$ or $u = \sin(4 \ \ -2)$ c) $u = \frac{3x}{x^2+3}$

AL-2. a) $u = y^{-5/2}$, $x = 1$, $y = 4^{-2/5}$, $u = 4$
b) $u = \sqrt{x^2 + 3x}$, $x = \frac{3 \pm \sqrt{409}}{2}$, $y = 1$, $u = 10$

AL-3. a) $u + \sqrt{u} - 6 = 0$ b) $v^2 + v - 6 = 0$ c) $v = -3$ or 2
d) $M = \frac{3 \pm \sqrt{29}}{2}$. No value associated with $v = -3$.

AL-4. $x + y$, $x^2 + 2xy + y^2$, $x^3 + 3x^2y + 3xy^2 + y^3$,
$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

AL-5. a) Decrease by 1 each time b) Increase by 1 each time
c) Each time the sum is the same as the exponent of expansion.

AL-6. a) $\frac{5}{2}$ b) $\frac{7}{3x+5}$ c) $\frac{2x+1}{x \pm 2}$ d) x^{-2} e) $x + 1$

AL-8. 1 goes in "Row 0."

AL-9. Row 9

AL-10. $x^9 + 9x^8y$

AL-11. a) 20 b) 5 c) 35 d) 252

AL-12. a) 1 b) 0 c) 4 d) 1
e) Impossible. $4^y \pi 0$ f) Impossible. $4^y \pi -3$

AL-13. a) $x^3 - 6x^2 + 9x + 3$ c) $W + 25$

AL-14. $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

AL-15. 1, 8, 28, 56, 70, 56, 28, 8, 1

AL-16. Multiply top and bottom by xy^2.

Al-17. a) $\frac{xy^3 + x^2y^2}{1 + x^3y^3}$ b) $\frac{x^2y^6 + 1}{1 + xy^3}$

AL-18. a) $\frac{x^2+y^2}{xy^2}$ b) $\frac{xy^2}{x^2+y^2}$ c) $\frac{x^2y^4}{(x^2+y^2)^2}$

AL-19. a) $\frac{y}{xy+1}$ b) $\frac{a^2b}{a^3+b^2}$ c) $\frac{a^4b^2}{(a^3+b^2)^2}$

AL-20. a) $x^2 + 3x^2y + 3xy^2 + y^2$ c) $x^4 + 12x^3w + 54x^2w^2 + 108xw^3 + 81w^4$

AL-22. $x^{15} + 15x^{14}y + 105x^{13}y^2 + 455x^{12}y^3$

AL-24. a) $x^3 + 3x^2 + 3x + 1$ b) $x^3 + 6x^2 + 12x + 8$

AL-25. a) $\frac{3+4x^4}{2x^4 \pm x}$ b) $\frac{y^2+x^2y^4}{x^4y^2 \pm x^2}$ c) $\frac{1+xy}{x^3y^2 \pm xy^2}$

AL-26. a) $x^3 + 3x^2y + 3xy^2 + y^3$ b) $y = -4w$
c) $x^3 + 3x^2(-4w) + 3x(-4w)^2 + (-4w)^3$ d) $x^3 - 12x^2w + 48xw^2 - 64w^3$

AL-27. a) xy^2 b) x^6y^{-1} c) $x^6y^{-2} + y^{-1}$

AL-28. a) 180° b) 225° c) -60° d) $-57.2957...^\circ$

AL-29. a) 9 b) 6 c) 60 units^3

AL-30. a) $\frac{}{6}, \frac{5}{6}, \frac{7}{6}, \frac{13}{6}$ b) same as part (a)

AL-31. a) $\left(\frac{7}{3}\right)$ b) $\left(\frac{x}{x+3}\frac{1}{}\right)$ c) $\left(\frac{x+5}{x+4}\right)$

AL-32. a) 1 b) -1 c) -1

PROBLEM SET A

1. $120\left(\frac{1}{2}\right)^{10}$ 0.117
2. $5(0.6)^4(0.4)$ 0.259
3. $(0.8)^4$ 0.410
4. $4\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)$ 0.422
5. $6(0.2)^2(0.8)^2$ 0.154
6. $20\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3$ 0.054
7. $153(0.9)^{16}(0.1)^2$ 0.284
8. $10(0.3)^2(0.7)^3$ 0.309

AL-35. a) $\frac{1}{16}$ b) Can get red-blue or blue-red combination.

AL-36. Total probability equals 1.
b) p^2 c) q^2 d) $2pq$

AL-37. a) $\sin^2 u - \sin u + 0.24 = 0$ b) Let $v = \sin u$
c) $v^2 - v + 0.24 = 0$ d) $v = \sin(3x - 5)$

AL-38. $a^4 + 4a^3bc + 6a^2b^2c^2 + 4ab^3c^3 + b^4c^4$

AL-39. b) $0.7^3 = 0.343$ c) $0.3^3 = 0.027$
d) $3(0.7)^2(0.3) = 0.441$ e) $3(0.7)(0.3)^2 = 0.189$

AL-40. a) $2 + 6\sqrt{3}$ b) 13 c) $b = 1$

AL-41. a) 207 b) 207 c) $2a^4 + 5a^2$
d) $2a^4 + 5a^2$ e) They are the same.

AL-42. $x = \frac{\sqrt{3} + 3\sqrt{2}}{^3+3}$ 0.176, $y = \frac{\sqrt{3} \quad ^3\sqrt{2}}{^3+3}$ -1.239

AL-43. a) any angle in the 4th quadrant b) $\frac{7}{6}$ or $\frac{11}{6}$
c) any angle in the 3rd quadrant
d) Approximately 2.8 radians works
e) No—it doesn't satisfy the Fundamental Pythagorean Identity.

AL-44. a) $6 - 2i$ b) $-2 + 18i$ c) $23 + 2i$

AL-45. $\frac{7}{41} + \frac{22}{41}i$

AL-46. a) $(x \pm y)(x + y)$ b) $\frac{(x\pm1)^2}{(x+1)}$ c) $\frac{1}{x^3\,(3x+y)}$

AL-47. One contains $x = 3$ and the other does not.

AL-48. a) $\frac{x+y}{x\pm y}$ b) $x \pm 4$ c) $\frac{(x+1)^2}{x+2}$

AL-49. $4 + 2\sqrt{3}$

AL-51. a) $7 + 3\sqrt{5}$ b) $\frac{11\pm3\sqrt{7}}{2}$ c) 17 $5\sqrt{11}$

AL-52. a) $\frac{a + 2\sqrt{ab} + b}{a \pm b}$ b) $\frac{a \quad b}{a \quad 2\sqrt{ab} + b}$

AL-53. a) $\frac{0}{0}$ indeterminate b) $\frac{1}{\sqrt{x+h} + \sqrt{x}}$ c) $\frac{1}{2\sqrt{x}}$

AL-54. $\frac{1}{\sqrt{3x+h} + \sqrt{3x}}$

AL-55. a) $\sqrt{3} \quad 1$ b) $\frac{x \pm \sqrt{x} \pm 2}{x \quad 4}$

AL-56. a) 8 b) -8 c) a^3 d) a^3 e) f(-a) = -f(a)

AL-58. a) $p^3 + 3p^2q + 3pq^2 + q^3$ b) $3pq^2$ c) q^3

AL-59. a) 7 + 3i b) 4 – 2i c) $\frac{2}{7} \pm \frac{i}{9}$

AL-60. a) $\frac{18}{25} \pm \frac{26}{25}i$ b) $3 \quad 7i$ c) -8i d) $\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$

AL-61. a) $\frac{p}{p^2q^2 \quad q}$ b) sin cos

AL-62. 13 + i

AL-63. a) $\frac{y^2 \quad x^2}{xy}$ b) $\frac{y^2 \quad x^2}{x^2y^2}$

AL-64 -2

AL-65. a) increasing $x > 2$; decreasing $x < 2$

AL-70. Increasing on (-• , -1) and (1, •), decreasing on (-1, 0) and (0, 1); concave up on (0, •), concave down on (-• , 0).

AL-71. a) decreasing (-• , 0) and (0, •), concave up (0, •), concave down (-• , 0)
b) increasing (-• , •), concave up (-• , •)
c) decreasing (-• , •), concave up (-• , •)
d) decreasing (-• , •), concave up (-• , 0), concave down (0, •)

AL-72. $x^3 – 12x$ is such a function.

AL-73. y = 5x, y = sin x, ...

AL-74. Shiloh; the segments are above the graph.

AL-75. It is the same.

AL-76. a) $x = -2 + 10i$ b) $x = 2 - i$
c) $x = 2 + 3i$ d) $x = 1.5 + 0.5i$

AL-77. $\frac{30+22\sqrt{2}}{17}$ b) $3\sqrt{2}+3$ c) $2+\sqrt{18}\pm\sqrt{12}+\sqrt{6}$

AL-78. a) -18 b) 18 c) $-a^3 + 3a$ d) $a^3 - 3a$
e) $f(-a) = -f(a)$

AL-79. a) $x^3 + 3x^2u + 3xu^2 + u^3$ b) $x^2 + 2yz + x^2$; $y^3 + 3y^2z + 3y^2 + z^3$
c) $x^3+3x^2y+3x^2z+6xyz+3xy^2+3xz^2+3y^2z+3yz^2+y^3+z^3$

AL-80. $x^4 + 4x^2 + 6 + 4x^{-2} + x^{-4}$

AL-81. b) It is a minimum because the rectangles are under the curve.
c) $0.4\sum_{j=0}^{4}\sqrt{(0.4j+1)^2+5}$

AL-82. a) $\frac{3x}{x^2\ y^2}$ b) $\frac{x^2+y^2}{x^2\ y^2}$

AL-83. b = 225.062

AL-84. -13

AL-85. a) 4, 4, 9, 9, 2.962, 2.962
b) Changing sign of x doesn't affect f(x). c) They are equal
d) $f(-a) = f(a)$ e) symmetric about y-axis

AL-86. a) even powers b) $f(-x) = f(x)$
c) They are symmetric about the y-axis.
d) $y = \cos x$ or $y = |x|$ are good choices.

AL-87. a) 8, -8, 27, -27, 5.097, -5.097
b) Changing the sign of x changes the sign of f(x).
c) opposite signs d) $f(-a) = -f(a)$
e) symmetric about the origin

AL-88. a) odd powers b) $f(-x) = -f(x)$
c) They are symmetric about the origin.
d) $y = \sin x$ is a good choice.

AL-89. a) (-2, 5) b) (3, -5) c) unknown

AL-92. b) $(x^{1/2})^6$ is not defined. Others are.
c) If x = -4, $x^{1/2} = 2i$ and $(x^{1/2})^6 = -64$

AL-93. C = 6 or 12

AL-94. a) 1 b) $\sqrt{12} = 2\sqrt{3}$

AL-95. a) 1 b) 4 c) 16 d) 25
e) $\frac{49}{4}$ f) $\frac{b^2}{4}$

AL-96. $y = (x + 1)^2$, $y = (x - 2)^2$, $y = (x + 4)^2$, $y = (x - 5)^2$,
$y = \left(x + \frac{7}{2}\right)^2$, $y = \left(x + \frac{b}{2}\right)^2$

AL-98. $y = (x + 3)^2 - 10$

AL-99. The pens should be 37.5 feet wide and 50 feet long.

AL-100. a) u = sin b) u = -1, 4. 0 is not in the domain. c) $= \frac{3}{2}$

AL-101. a) 16 b) 16 c) 32 d) $\frac{1}{16}$

AL-102. a) 2 b) $7^{1/60}$

AL-103. b) sine is odd, cosine is even

AL-104. a) even b) neither c) odd

AL-105. a) 9 b) $y = (x + 3)^2 - 6$ c) V(-3, -6)

AL-106. V(4, -15)

AL-107. a) 27 b) $y = 3(x - 3)^2 - 26$, V = (3, -26)

AL-108. a) $y = 2(x - 2)^2 - 1$ b) (2, -1)

AL-109. $(x + 3)^2 + (y - 2)^2 = 64$; center: (-3, 2); radius: 8

AL-110. c) $S = 2x^2 + 6xh$; $4500 = 2x^2h$ e) $h = \frac{2250}{x^2}$, $S = 2x^2 + \frac{13500}{x}$
f) $x = 15$, $h = 10$, $S = 1350$

AL-111. The only change is that $S = 4x^2 + 6xh$. After solving, $x = \frac{15}{\sqrt[3]{2}}$ 11.906, $h = 10\sqrt[3]{4}$ 15.874, $S = 900(4^{1/3} + 2^{1/3})$ 1700.893

AL-112. center: (-4, 9), radius: $\sqrt{120}$ 10.954

AL-113. $a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4$

AL-114. No, because the vertex must lie on the y-axis if it is even.

AL-115. $x = \sqrt{2}$ 1

AL-116. a) even b) neither c) odd

AL-117. a) 2 b) -1 c) $\sqrt{10}$
d) 2 e) 7 f) $\frac{1}{4}$

AL-118. They're the same because $60° = \frac{}{3}$ radians.

AL-119. a) C = (5, -4), r = 6 b) C = (4, -3), r = 9, shaded on inside

AL-120. a) $x^2 + y^2 = 100$ b) $(x - 7)^2 + (y - 5)^2 = 65$

AL-122. $2x^2 - 2x + 2$, remainder -5 or $2x^2 - 2x + 2 - \frac{5}{2x+1}$

AL-123. $2x^2 - 4x + 13$

AL-124. It is. The quotient is $x^2 - 6x + 9$ with no remainder.

AL-125. a) $f^{-1}(x) = x^3 - 2$
c) $f^{-1}(x)$ is a reflection of $f(x)$ over the line $y = x$.

AL-126. $g^{-1}(x) = \frac{2x}{x \pm 2}$ or $\frac{2x}{2 \quad x}$

AL-127. $f(x) = (x - 6)^{1/3}$

AL-128. It is not. Remainder is 140.

AL-129. $f^{-1}(x) = \frac{x}{1\ x}$ or $\frac{x}{x\ 1}$

AL-130. b) It won't pass the vertical line test. d) $x \geq 0$
e) $f^{-1}(x) = \sqrt{\frac{2x}{1\ x}}$

AL-131. $8 - 36x + 54x^2 - 27x^3$

AL-132. a) $x^3 + 6x^2y + 12xy^2 + 8y^3$ b) $x^3 - 6x^2y + 12xy^2 - 8y^3$
c) Every other sign is different. $(-1)^k$ changes every other sign.

AL-133. $x = \pm i\sqrt{\frac{1}{3}}$, $y = \frac{5}{3}$

AL-134. a) 2.25 b) $\frac{42.4}{b^8}$

AL-135. b 0.709, a = 329.28

AL-136. d; p = 0.4 or 0.779. Teacher Solution: The probability of exactly three heads is $\binom{5}{3}p^3(1-p)^2$ which we want to equal $\frac{144}{625}$. That is, we need to solve the equation $10p^3(1-p)^2 = \frac{144}{625}$. Solving graphically gives the required solutions.

AL-137. $\frac{x^4+1}{x+x^2}$

AL-140. b) 2 d) 70

AL-141. Pascal's Triangle

AL-145. a) $\sqrt{2}:1$ b) $x = \frac{2}{\sqrt{2}\pm 1}$ c) $x = 2(\sqrt{2}+1)$

AL-147. Let $x = a + bi$ and so $x^2 = a^2 - b^2 + 2abi = i$. Thus $2ab = 1$ and $a^2 - b^2 = 0$. Thus $x = \pm\frac{1+i}{\sqrt{2}}$. Students that get seriously involved in this problem might want to be told eventually about the fact that $e^{i} = \cos + i\sin$ which is an easy way to take roots.

AL-148. It is a line.

AL-149. a) $\sqrt{8}$ 2 b) $\frac{4}{2+\sqrt{8}} = \frac{2}{1+\sqrt{2}}$

AL-150. a) $\frac{x^4y^6+y^3}{x^2+x^3y^3}$ b) x

AL-151. It would not be a function because it fails the Vertical Line Test.

AL-152. Error is here. Parentheses should contain $x^2 - 2x$.

AL-153. a) a parabola with a vertex on the y-axis b) impossible
c) a parabola with a vertex not on the y-axis

AL-154. odd

AL-156. C = 6 or 12

AL-157. a) $\frac{1}{2}$ b) $\frac{\sqrt{2}}{2}$ c) -1 d) 0

AL-158. a) $\frac{2}{3}, \frac{5}{3}$ b) $\frac{5}{6}, \frac{7}{6}$ c) $\frac{}{6}, \frac{5}{6}, \frac{7}{6}, \frac{11}{6}$

