

# Chapter 7

## The New Pilot: VECTORS AND MORE TRIGONOMETRY

- NP-1. a)  $K = \frac{1}{2} ah$     b)  $h = b \sin C$     c)  $K = \frac{1}{2} ab \sin C$
- NP-2.  $11.644 \text{ cm}^2$     NP-3.  $K = \frac{1}{2} bc \sin A$  and  $K = \frac{1}{2} ac \sin B$
- NP-4.  $128.558 \text{ sq. ft.}$
- NP-5. a)  $\frac{2}{3}, \frac{4}{3}, \frac{2}{3}, \frac{3}{2}$     b)  $0, 2, \frac{7}{6}, \frac{11}{6}, \frac{3}{2}$   
 c)  $\frac{2}{3}, \frac{4}{3}, \dots, \text{all } + 2n$     d)  $\frac{5}{6}, \frac{5}{6}, \frac{3}{2}, \text{all } + 2n$
- NP-6.  $\frac{2}{6} + \frac{2}{3}n$     NP-7. 0.122 inch
- NP-8. a)  $\sin 10$     b)  $\sin(x + y)$     c)  $\cos x \cos h - \sin x \sin hd) \cos 6$
- NP-9.  $3 + 4i, -3 + 4i$     NP-10.  $y = 16.98(3.404)^x$
- NP-11. a)  $-1 \leq x \leq 2, 0 \leq y \leq 1$     b) shift 3 units left    c) stretch vertically
- NP-12.  $\sqrt{138}$  11.747 feet    NP-13. a)  $\frac{a}{c}$     b)  $\frac{c}{b}$     c)  $\frac{b}{a}$
- NP-14. a) 5    b)  $16 - 8 \cos C + \cos^2 C$
- NP-15. a)  $h = b \sin A$     b)  $h = a \sin B$     d)
- NP-16. b) Solve  $\sin A = \frac{h}{b}$  for  $h$ .    c) yes  
 d) The same two equations are true as in the acute case, so the algebra proceeds as before.    d) yes
- NP-17. a)  $5 \sin R$     b)  $5 \cos R$     c)  $7 - 5 \cos R$   
 d)  $(5 \sin R)^2 + (7 - 5 \cos R)^2 = 6^2$     e)  $\cos R = \frac{19}{35}, R = 57.1^\circ$   
 f)  $P = 44.4^\circ, Q = 78.5^\circ$
- NP-19. a) 184.018 ft    b) 294.354 ft

NP-20. Can't in (a) and (b) because you'll get two unknowns in any form of the equation. Can't in (d) for the same reason, and also because the triangle is not determined. Note that (c) is the only diagram in which you're given exactly one side

NP-21. a)  $G = 78^\circ$ ,  $OG = 7.351$  in,  $DG = 5.035$  in    b) 18.102 sq. in

NP-22. a)  $x^5 - 1$     b)  $x^2 + x + 1$     NP-23. a)  $70^\circ$  b)  $95^\circ$

NP-24. a)  $x = \sqrt[6]{200}$     b)  $x = \log_6 200$     c) 2.418, 2.957  
d) 0.243, 18.361,  $6^6$

NP-25. a)  $\frac{5}{6}, \frac{5}{6}, \frac{3}{2}, \frac{3}{2}, \text{all} + 2$  n    b) 0.644, 5.640, , all + 2 n

NP-26.  $-\frac{1}{x}$     NP-27.  $16218 x^{-1.5}$     NP-29.  $a^2 + b^2 - 2ab \cos$

NP-30. a)  $a \cos C$     b)  $a \sin C$     c)  $b - a \cos C$   
d)  $c^2 = (a \sin C)^2 + (b - a \cos C)^2$

NP-31.  $36.3^\circ$

NP-32. a) Yes, the third side is 142.6 ft    b)  $47.9^\circ$  and  $58.1^\circ$   
c) Yes, because the lot area is approximately 6661 square feet

NP-33. Can't in (c) or (d) because you'll get two unknowns in any form of the equation. Also, the triangle is not determined in (d).

NP-35.  $x = 4, y = \pm 2\sqrt{2}$     NP-36.  $\frac{x-1}{(x+2)(x+1)}$

NP-37. a) Teacher Solution: This can be done using a combination of the Law of Cosines and the Law of Sines or else by observing that Ali is at  $(200 \cos 15^\circ, 200 \sin 15^\circ)$  and so the angle is  $15^\circ$  more than

$$\tan^{-1}\left(\frac{100 - 200 \sin 15^\circ}{200 \cos 15^\circ}\right) = 14.109$$

b)  $\tan^{-1}\left(\frac{100 - x \sin 15^\circ}{x \cos 15^\circ}\right)$

NP-39. center at  $(-3, 2)$ , radius = 5    NP-40.  $y = \frac{19.2}{x^2}, g(6) = \frac{8}{15}, g(-3) = 2\frac{2}{15}$

NP-41.  $f^{-1}(x) = \frac{2x-2}{2}$     NP-42. 12.393 cm    NP-43.  $14 < x < 70$  inches

- NP-44. The Law of Sines calculation results in the sine of the angle at Icy's being greater than 1. The Law of Cosines calculation yields a quadratic equation with no real solutions.
- NP-45. a)  $I = 44.8^\circ$  (or  $I = 135.2^\circ$ , but don't point this out yet)  
 b) The answer students will get is  $d = 40.69$  m (and  $D = 107.2^\circ$ ).  
 c) Katya missed the possibility that  $I$  could be obtuse. In fact,  
 $I = 135.2^\circ$ ,  $D = 16.8^\circ$  and  $d = 12.29$  m.
- NP-46. a)  $a = 5$                       b)  $C = 90^\circ$                       d)  $C = 45.6^\circ$  or  $C = 134.4^\circ$   
 e)  $ACB = 180^\circ - BCC' = 180^\circ - BC'C$  since  $BCC'$  is isosceles.  
 f) Supplementary angles have the same sine.                      g) one  
 Challenge) 0 triangles if  $a < c \sin A$ ; 1 triangle if  $a = c \sin A$  or  $a \geq c$   
 2 triangles if  $c \sin A < a < c$ .
- NP-48. a)  $-1 < x < \frac{2}{3}$                       b)  $x = -2$  or  $x \geq \frac{1}{3}$
- NP-49. b) 1.428                      c) 2.059                      d) 32 micrograms
- NP-50.  $b = 5\sqrt{3} \pm 2\sqrt{6}$                       NP-51.  $h = 3779.11$  feet
- NP-52. a)  $92.87^\circ, 48.52^\circ, 38.62^\circ$                       b) 14.984 square meters
- NP-53.  $x = 0, -\frac{5}{2}$                       NP-54. a)  $-2x + h$                       b)  $-4y^2(x^2 + y^2)^{1/2}$
- NP-55. a)  $x \pi - 1$                       b) It approaches  $\bullet$ .                      c) It approaches  $-\bullet$   
 d) It approaches 3.                      e) It approaches 3.
- NP-56.  $\cos C = 0$ ;  $c^2 = a^2 + b^2 - 2ab(0)$ ;  $c^2 = a^2 + b^2$                       NP-57.  $c = 12.490$
- NP-58. a)  $90^\circ$                       b)  $225^\circ$                       c)  $170^\circ$
- NP-61. a) yes                      b) no                      d) e                      e) g  
 f) c                      g) d and e                      h) i and j  
 i) i and d; i and e; g and c; h and f  
 j) Order is not important. Vector addition is commutative
- NP-63. b)  $247\infty$                       c)  $345\infty$
- NP-64. a) vectors  $s$  and  $r$ , vectors  $l$  and  $k$                       b) yes

- NP-66. c) Go 8 steps at a bearing of  $300^\circ$  NP-67. d) parallelogram
- NP-68. a)  $2^{21}$  b)  $x^3$  c)  $x^{12}$  d)  $m^{-10}$
- NP-69.  $x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$  NP-70. a) 0.412, 2.73 b)  $\frac{2}{3}, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}$
- NP-71.  $y = 56.234(6606.934)^x$  NP-72. c NP-73.  $22\sqrt{2}$  at  $135^\circ$
- NP-74. 8.66 due east, 5 due north. NP-76. b) (-4, 3)
- NP-77. a)  $6i - 2j$  b) (-1, 3) c)  $2i$
- NP-78. b)  $-25\sqrt{2}i - 25\sqrt{2}j$  c) the horizontal component  
d) EMBED "Equation" \\* mergeformat  $25\sqrt{2}$  35.355 lbs
- NP-79. a) 5 b) Because the resultant is 1 unit long. c)  $\frac{3}{5}i + \frac{4}{5}j$
- NP-80. a)  $2a$  b)  $0a$  is equivalent to 0;  $-1a$  is equivalent to  $-a$ .  
c) opposite direction
- NP-81. a)  $a - b = a + (-b)$
- NP-82. b) Use a vector equivalent to  $b$  which begins at the end point of  $a$ .  $a + b$  is then the vector from the initial point of  $a$  to the end point of  $b$ .  
c)  $3i + 2j$  d)  $b + c = -2i + j$ ,  $c - a = -5i - j$
- NP-84. a) (5, 0). There are many other answers.  
b) (6, 8). There are many other answers.
- NP-85.  $r = (0, -7) = -7j$ ,  $s = (0, 3) = 3j$ ,  $u = (-3, 5) = -3i + 5j$ ,  
 $v = (8, -4) = 8i - 4j$ ,  $z = (-3, 5) = -3i - 5j$   
a)  $8i - 4j$  b)  $-6i$  c)  $4i - 2j$  d)  $-11i + 9j$
- NP-86. b)  $\frac{5}{13}i + \frac{12}{13}j$   
c) Divide the coefficients of  $i$  and  $j$  by the magnitude of the vector.
- NP-87. a) a force, weight, wind c) 35 mph (no direction mentioned)d) straight down  
the vector points straight down. The picture is clearest  
if the vector begins at the book.
- NP-88. a) 15 b) 1 c) 222 d) 7  
e)  $a^2 + 6a + 6$  f)  $-a^2 + 2a + 8$  g) They are not equal.

NP-89. Magnitude =  $\sqrt{34}$  5.831. Bearing =  $211.0^\circ$

NP-90.  $-21.213i + 21.213j$

## NP-91. PROBLEM SET A

- |                                       |   |
|---------------------------------------|---|
| 1. 5                                  | 2. -1   |
| 3. 0; orthogonal                      | 4. 0; orthogonal                                    |
| 5. 8                                  | 6. 7  |
| 7. $2x + 6$ ; orthogonal if $x = -3$  | 8. $-7x + 6y$ ; orthogonal if $y = \frac{7}{6}x$    |
| 9. $7x$ ; orthogonal if $x = 0$       | 10. $x^2 + y^2 - 4$ ; orthogonal if $x^2 + y^2 = 4$ |
| 11. $19.654^\circ$ or $0.343$ radians | 12. $107.1^\circ$ or $1.869$ radians                |
| 13. $42.397^\circ$ or $0.740$ radians | 14. $81.870^\circ$ or $1.429$ radians               |
| 15. $69.124^\circ$ or $1.206$ radians | 16. $90^\circ$ or $1.571$ radians                   |
| 17. 4                                 | 18. -10   |
|                                       | 19. $\frac{3}{5}$                                   |
| 20. 1.5                               | 21. -2  |
| 23. -0.5                              | 22. 1.5   |
|                                       | 24. $\frac{26}{7}$                                  |

## PROBLEM SET B

- |                       |                                      |                     |
|-----------------------|--------------------------------------|---------------------|
| 25. (0.6, 0.8)        | 26. $\frac{14}{3}$                   | 27. $\pm 2\sqrt{3}$ |
| 28. $z = \pm\sqrt{6}$ | 29. $u = \frac{v}{\sqrt{v \cdot v}}$ |                     |

## PROBLEM SET C

- |                        |                               |
|------------------------|-------------------------------|
| 31. $x = -1, y = -6.5$ | 32. $\ p + q\  = \ p - q\ $ . |
|------------------------|-------------------------------|

- |                                       |                |               |
|---------------------------------------|----------------|---------------|
| NP-92. a) $105^\circ$                 | b) $137^\circ$ | c) $783.23$ m |
| d) $68.0^\circ$ ( $BAC = 7.0^\circ$ ) |                |               |

- |  |                                   |
|--|-----------------------------------|
| NP-93. a) $783.23$ mph                 | b) $68.0^\circ$                   |
| c) Virtually the same as the last one. | d) $68.0^\circ$ ; $1566.46$ miles |

- |                      |  |      |
|----------------------|--|------|
| NP-94. a) $45^\circ$ | b) $70.848$ mph, angle opposite the "64" side = $15.4^\circ$ so the bearing is $254.6^\circ$ | c) 1 |
|----------------------|--|------|

NP-95. a) 96 miles SE of Houston      b) Fly with a bearing of  $347.6^\circ$ . The entire trip takes about 2 hours, 27 minutes.  
c)  $282.4^\circ$  1.971 hours or 1 hour, 58.3 minutes

NP-96. a) Mag. =  $3\sqrt{2}$  4.24,  $\theta = \frac{7}{4}$   
b) Mag. =  $\sqrt{145}$  12.04,  $\theta = \tan^{-1}(-12) + \pi = 1.654$

NP-97.  $15 \sin 20^\circ = 5.13$  km/hr

NP-98. a)  $85^\circ$       b) 129.81 mph      NP-99.  $h = \frac{36}{20}d$   
c)  $127.6^\circ$  (Angle opposite 110 side =  $27.4^\circ$ )

NP-100. a)  $(2.5\sqrt{3}, 2.5)$       b)  $(-5\sqrt{2}, -5\sqrt{2})$       c)  $(-7.5, 7.5\sqrt{3})$

NP-101. a)  $(-2, 4)$       b)  $-10i - j$       c)  $(0, 0)$

NP-102. a)  $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}$       b)  $0.723, 5.56, \frac{2}{3}, \frac{4}{3}$

NP-104. a)  $(-32.14, 38.30)$       b)  $(121.24, -70)$       c)  $(-24.75, 24.75)$

NP-105.  $(89.1, -31.7)$ : Magnitude 94.57 at bearing  $109.6^\circ$

NP-106. a)  $-2i + 12j + 14k$       b)  $-5i - 4j$

NP-107. b)  $AB = 6i - 3j$ ,  $C = (2, 2.5)$       c)  $D = (3, 2)$   
d)  $E = (5, 1)$ ,  $F = (8, -.5)$ ,  $G = (-7, 7)$

NP-109. a)  $P = A$       b)  $P = B$       c) between A and B      d) beyond B  
e) beyond A

NP-110. a)  $(-5, -7)$       b)  $(-2, 3) + t(-5, -7)$

NP-111. b)  $(1, 2, 4) + t((-2, 1, 2))$

NP-112. a)  $152^\circ$   
b) 302.97 mph, angle opposite the "80" side =  $7.1^\circ$   $103.9^\circ$

NP-113.  $13i - 8j - 20k$       NP-114. 179.6 mph at a bearing of  $99.2^\circ$

NP-115.  $2i + t(2i - j)$  NP-116. c) no .

Challenge. You must pull with  $40 \sin 22^\circ \sec 16^\circ = 15.588$  pounds of force.

NP-118. a) Not enough information for a specific time. All we know is the average rate at that time. b) 55.8 mph

NP-119. a) 1 b) 1 c)  $-\sin^2(2x + 3)$  d)  $\sin 2$

NP-120.  $f(x) = 1$  if  $x > 0$ ,  $-1$  if  $x < 0$

NP-121 a)  $(-8, 12)$  b)  $-7i + 5j$  c)  $-5i + 6j$  NP-122. 11.619 sq. ft

NP-124. a)  $(0, -15), (34.64, 20), (-21.21, 21.21)$  b)  $(13.43, 26.21)$   
 c) 29.45 mph, 62.9 $\infty$ upstream d) 1 hour, 29.4 minutes  
 e) 39.1 miles

NP-125. a)  $52.3^\circ, 56.1^\circ, 71.6^\circ$  b) 49.818 sq. km

NP-126.  $81.2^\circ, 223.35$  mph

NP-127. a) -37 b) 134.2 $\infty$  c)  $(-6, -4)$   
 d)  $(-2, 5) + t(-2, -14)$

NP-128. a) 72.6 $\infty$ downstream at 9.25 ft/s b) 36.2 s c) 319.5 ft.

NP-129.  $2i + j + 3k + t(2i - 3j + 2k)$

NP-130. a) v and vi b) i and iii c) v

NP-131. a)  $\frac{20}{3}$  b)  $4 + 2 \cdot \frac{1}{2} \cdot 2.5 + 2 \cdot \frac{1}{2} \cdot 2 \cdot \sqrt{29} = 17.27$   
 c)  $= \cos^{-1} \frac{[(2,0,5) \cdot (2,2,5)]}{\sqrt{29} \sqrt{33}} = \cos^{-1} \frac{\sqrt{29}}{\sqrt{33}} = 20.4^\circ$

NP-132. a)  $5\sqrt{2}$  b)  $EF = 5$  cm;  $DF = 5\sqrt{3}$  cm NP-133. 200 ft

NP-134. a)  $\cos$  b)  $\sec 2x$

NP-135. a) even b)  $f(x)$  gets bigger. c)  $f(x)$  gets bigger. d)  $x = \pm \sqrt{3}$

NP-136.  $g(x)$  increases to  $\bullet$ ;  $g(x)$  decreases to  $-\bullet$