Chapter 8

To Infinity and Beyond: LIMITS

LM-2. f) If the procedures are followed accurately, all the last acute angles should be very close to 60 degrees.

LM-3. They should say 60∞ Once this angle is reached, the obtuse angle will be 120∞ This is, of course, interesting since theoretically this angle is never reached. When it is bisected the resulting angle will equal 60∞ Do not simply accept as justification that the pattern continues.

LM-4. b) 20 (given), 80, 50, 65, 57.5, 61.25, 59.375, ...
c) 60.000 is obtained by n = 18. Students can repeat this process by using the ANS key on their calculator. Enter 20 and push EXE or ENTER. At the next line use (180 – Ans)/2. Repeatedly hitting EXE or ENTER will generate the sequence.
d) t(n) approaches 60.
e) It has a horizontal asymptote at 60. Some students are uncomfortable with an asymptote that gets crossed. Reassure them it is okay.

LM-5. a) 40, -20, 10, -5, 2.5, -1.25, 0.625, ...
b) geometric sequence $r = -\frac{1}{2}$.
c) The error gets closer and closer to 0.

LM-6. f(x) is even: • , • . If f(x) is odd: • , -•

LM-7. a) 0 b) • c) 0
d) No limit. Sin x bounces around and does not approach any number.

LM-8. a) $p_2 = -5$, $p_3 = 7$
b) n = 4, $p_0 = 0$, $p_1 = 7$, $p_2 = -9$, $p_3 = 0$, and $p_4 = 2$
c) n = 8, $p_8 = 12$, $p_4 = 3$, $p_0 = -10$, $p_n = 0$ for n = 1, 2, 3, 5, 6, 7

LM-9. Left Column: 0, • . Right Column: • , 0

LM-10. a) The limit is 60; it will never actually equal 60 unless the original acute angle was 60∞
b) No, because the points alternate above and below 60.

LM-11. b) 16 + 10 + 24 = 50 c) 110 LM-12. a) 0 b) •

LM-13. a) $\boxed{\frac{4}{5}}$ b) 65.974 c) 76 d) 13.572

LM-14. a) Add 4. b) Subtract 5. c) Add 1. LM-15. $3x + 2y = 31$

LM-16. a) $x = 0, y = 0$ b) $y = 0$ c) $x = -2, y = 0$ d) $y = -3$

LM-17. a) $\boxed{\frac{3}{x} + 60}$ b) $-\frac{2}{x} \pm 40$ c) $\boxed{\frac{1}{5x} + 4}$ d) $\boxed{\frac{5}{x} + 20}$

LM-18. a) 0 b) • c) 3 LM-19. a) • b) 0 c) • d) •

LM-20. a) • b) -• c) -• d) •

LM-21. a) The dominant term is -2 y 5 b) 20 x 9
c) x 6 d) x

LM-22. a) • b) • c) -• d) -• LM-23. All are • .

LM-25. a) 60 b) -40 c) 2 d) 20

LM-26. a) 110, 430 b) -4, 12, 140, 19596
c) $\frac{1}{4}$, 4, $\frac{1}{4}$, 4 d) 15, 75, 1125, 84375

LM-27. The end behavior of the graph.

LM-28. a) • b) • c) -• d) -•

LM-29. The larger base will determine if the function goes to • or -• .

LM-30. The limit = 0. As x goes to • , x^n goes to • . As the denominator gets large and the numerator remains constant, the fraction will get closer to zero.

LM-31. a) 0 b) 3 c) 12

LM-32. a) $x^2 + y^2 = 169$ b) 247.4˚ or -112.6˚ d) $\frac{144}{169}$ 0.85

LM-33. $\frac{13}{12}$ LM-34. 5, 7, 11, 17, 25

LM-35. a) 0 b) 2.5 c) •

LM-36. a) 2 b) 2 c) 2
e) From LM-31, students should see that c(x) is the easiest.

LM-37. a) 0.5 b) 0 c) $\bullet$

LM-38. a) The limit is 0. b) The limit will be $\bullet$ or $-\bullet$.
c) The limit will be the ratio of the coefficients of the dominant terms.

LM-39. The limit = 3. LM-40. a) 5 b) log 3

LM-41. a) $x \log 2 - 5 \log x$ $0.301x - 5 \log x$ b) Both are infinite.

LM-42. a) 2, -2 LM-43. a) 0 b) -0.5 c) no limit

LM-44. a) $S = 6L^2$ b) $D = L\sqrt{\mathbf{3}}$ c) $D = \sqrt{\frac{S}{2}}$

LM-45. b) initial value = 1.87, multiplier = 1.02, time is in months.
c) initial value = 1000, multiplier = 1.3, time is in hours.
d) initial value = 5000, multiplier = 1.01, time is in months.

LM-46. a) \$5634.13 b) \$5412.16 c) 5266.74 d) 5378.61

LM-47. a) $\sum_{k=0}^{10} \frac{1}{k!} = 2.71828$, $\sum_{k=0}^{4} \frac{1}{k!} = 2.708333$ b) *e*

LM-48. a) increasing b) decreasing c) decreasing

LM-49. 4 LM-50. $y = \frac{3}{2}x + 2$

LM-51. b) $y = 3 + 0.01t$ d) $3T + 0.005\ T^2$

LM-52. a) Add 4 or $t(n) = 4n - 1$
b) The function includes all numbers, while the sequence only contains whole number values of n

LM-53. a) 2.594 b) 2.705 c) 2.717 d) 2.718

LM-54. a) \$2250 b) \$2441.41 c) \$2613.04 d) \$2704.81
e) $\boxed{\$1000\left(1+\frac{1}{n}\right)^n}$

LM-55. The amount gets close to \$2718.28.

LM-56. Answers are \$1562.50, \$1601.81, \$1632.09, \$1646.67, $\$1000\left(1+\frac{0.5}{n}\right)^n$, and \$1648.72.

LM-57. b) $1000e$ c) e LM-58. b) 2.302, 4.605, 0, 1.609, 1

LM-59. e LM-60. a) 0 b) no limit
c) If n is an integer, sin n = 0, but sin x oscillates between -1 and 1 as x •.

LM-61. a) $\boxed{\frac{2}{3}}$ b) no limit c) 0 d) -•

LM-62. a) \$116.18 b) 13.863 yrs c) 13.863 yrs

LM-64. a) \$39.34 b) \$40.28 c) This is a matter of opinion.

LM-65. a) $\frac{\ln 2}{r}$ b) R = 100r
d) We use 70 because it makes the mental arithmetic easier. e) yes

LM-66. a) It approaches zero. b) We have a horizontal asymtote at y = 0.
c) Approaches -• , approaches • . d) error; vertical asymptote.
e) f(x) = $-\frac{1}{64}$; a point at $(-2, -\frac{1}{64})$
f) As x -2, f(x) $\frac{1}{64}$. As x 2^-, f(x) -•
g) The superscript signs in x $\bullet^-$ and x $-\bullet^+$ are redundant.
x $\bullet^+$ is impossible.

LM-67. a) f(x) 1 b) f(x) • c) f(x) •
d) f(x) $\frac{37}{36}$ e) undefined f) f(x) $\frac{37}{36}$

LM-68. 2x – 3y = 2 LM-69. a) 0 b) 0 c) 3 d) 3

LM-70. a) x – 1 b) x – 3 c) $\boxed{\frac{x(x+3)}{x+4}}$ d) -(x + 3)

LM-71. a) x π -3 b) x π 3 and x π -3 c) x π 3 and x π -3

LM-72. a) f: x = -1, g: x = -2
d) The function is not defined—vertical asymptote.

LM-73. They have similar form; domain is all real numbers except one point.

LM-74. f(x) has a vertical asymptote g(x) has a hole. Common factor in numerator and denominator.

LM-75. hole at x = -2,
asymptotes: vertical at x = -3 and x = -4; horizontal at y = 0,
horizontal asymptote at y = 0

LM-77. Answers will vary. Examples are given.

a) $\frac{1}{x \pm 3}$ b) $\frac{x^2 \pm 9}{x \pm 3}$ c) $\frac{x^3 \pm x}{x^2 \pm 1}$ d) $\frac{x^2 + 2x \pm 24}{x \pm 4}$

e) $\frac{x \pm 2}{x^2 + x \pm 6}$ f) $\frac{(x+2)(x^2 \pm 9)}{x+2}$

LM-78. a) No; f(-3) and the limit do not exist. b) continuous
c) No, limit $\neq$ f(2). d) f(4) does not exist.

LM-79. a) 0.5 b) 0 c) $\bullet$

LM-80. a) 73.655 b) 2.047 LM-81. 1, 4, 9, 16, 25

LM-82. 0, -3, -2, -1 LM-83. At t = 1 it has a height of 16 ft.

LM-84. $-\sqrt{3}$ LM-85. 4i + 2j + t(-6i - 3j) LM-86. a) 4 b) 64

LM-88. a) 0 b) $\frac{2}{3}$ c) $\frac{7}{5}$ LM-89. a) Areas are the same.

LM-90. b) Each sector is smaller and the resulting figure will be more rectangular in shape.
c) The base becomes flatter. It approaches a segment.
d) They should answer a parallelogram or a rectangle.
e) $\frac{1}{2}$ circumference; rectangle

LM-91. b) area = r^2 b) Answers will vary.

LM-92. a) 6 b) 7 c) no limit d) 6 e) $-\bullet$
f) $-\bullet$ g) No, it is not continuous at x = 3.

LM-93. a) 0, no limit b) 0, no limit c) y = 0 for both

LM-94. a) -4 b) 1.956 c) 2 d) (a) and (c)

LM-95. a) 0 b) 0 c) • d) 0

LM-96. Examples: $f(x) = \frac{x \pm 3}{x(x \pm 3)}$, $g(x) = \frac{x \pm 3}{x^2(x \pm 3)}$

LM-97. 1.618033989. No—it reaches this value by the 25th term. The limit of the calculator's display is reached by this point so it's forced to round off.

LM-98. $10 - 8\sqrt[3]{2}$ -0.079 LM-99. x = 9, y = -18, z = -21

LM-100. a) $\boxed{4x + 4\sqrt{25 \pm x^2}}$ b) $\boxed{4x\sqrt{25 \pm x^2}}$ LM-101. 75 mph

LM-102. a) e^3 b) ln 200 c) $e^4 - 1$ d) ln 80 – 2

LM-103. a) 17, 20, 23 b) 162, 486, 1458 c) 21, 34, 55

LM-104. a) 2, 3, 5, 9, 17 b) 1, 3, 7, 13, 21 c) 1, 1, 2, 3, 5

LM-105.

Month	Baby Rabbits	Teen rabbits	Parent rabbits	Total rabbits
1	1	0	0	1
2	0	1	0	1
3	1	0	1	2
4	1	1	1	3
5	2	1	2	5
6	3	2	3	8
7	5	3	5	13
8	8	5	8	21
9	13	8	13	34
10	21	13	21	55
11	34	21	34	89
12	55	34	55	144

LM-106. a) 8, 13, 21, 34, 55, 89, 144 b) $a_n + a_n + 1$

LM-107. b) yes; 1.618 LM-108. b) yes; 0.618 LM-109. Amy is correct.

LM-110. i) f has a vert. asym. at x = 3 and x = -1, root at x = 1, and hole at x = -2.
ii) g has a vert. asym. at x = c, root at x = -c, and holes at x = a and x = b.
iii) h is the same as g.

LM-111. a) 4 b) 4 c) 2 d) 2 e) $-\infty$
e) $-\infty$ f) ∞ g) yes

LM-112. a) 1, 1, 1, 1, 1 b) 2, -1, 11, 71, 4691

LM-113. The only two values are 1 and 5. LM-114. 56 quarters, 14 years

LM-115. a) 3.33333 . . . c) 9N = 3 d) N = $\frac{1}{3}$

LM-116. a) $\boxed{\frac{5}{9}}$ b) $\boxed{\frac{7}{9}}$

LM-117. a) Divide by 10 or multiply by 0.1. b) $a_n = \frac{1}{10^n}$ or $(0.1)^n$
c) 0.1111 . . . d) $\boxed{\frac{1}{9}}$

LM-118. 100, $\boxed{\frac{4}{11}}$ LM-119. a) $\frac{20}{33}$ b) $\frac{37}{33}$ c) $\frac{41}{333}$

LM-121. $\boxed{\frac{1\pm\sqrt{5}}{2}}$ LM-122. a) 0 b) $\frac{7}{4}$ c) $\boxed{\frac{31}{16}}$ d) 2

LM-123. e) i) 1.618 ii) 0.618

LM-124. b) $L - \frac{1}{L} = 1$ c) $\frac{1\pm\sqrt{5}}{2} = 1.618$ or -0.618. We use the first.

LM-125. a) It goes to infinity. b) $r \geq 1$ c) $\lim_{n\to\infty} a_n = 0$

Sequence	Sum	$\lim_{n\to\infty} a_n$
1, 2, 4, . . .	∞	∞
2, 2, 2, . . .	∞	2
12, 18, 27, . . .	∞	∞

LM-126. a) $|r| < 1$ b) $ar + ar^2 + ar^3 + ar^4 + \ldots$
c) a d) $S = \frac{a}{1-r}$

LM-127. a) $\boxed{\frac{10}{9}}$ b) 121.5 c) ∞ d) $\boxed{\frac{40}{3}}$

LM-128. The sum diverges to ∞ for positive values of r and has no limit for negative values of r.

LM-129. $|r| < 1$. If $r > 1$, the sequence terms go to ∞ or $-\infty$. If $r \leq -1$, the series has no limit.

LM-131. b) 1.618 c) 0.618

LM-133. a) It is the common ratio or multiplier for the sequence.
b) 3(486) c) 2S = 1456 d) S = 728

LM-134. a) 635 b) $\boxed{\frac{6(1-x^5)}{1-x}}$ c) 20802.54 d) $\frac{a(1-r^{12})}{1-r}$

LM-135. a) $\frac{a(1-r^{n+1})}{1-r}$ b) $\frac{a}{1-r}$ LM-136. 1, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$

LM-137. a) 2 b) $\frac{64}{3}$ c) 10a d) ∞, because $r > 1$

LM-138. a) no value b) no value c) 1 d) 1
e) $\frac{1}{2}$ f) $\frac{1}{2}$

LM-140. a) 11.625 b) 11.985 c) 12

LM-141. $6\sum_{k=0}^{12}(0.5)^k$ LM-142. $a_{n+1} = 0.5a_n + 6$, $a_o = 6$

LM-143. 12 mg

LM-144.

Time units (8 hr interval)	Amount of antibiotic	Expanded amount
0	6	$0.5^0\ 6$
1	0.5 (6) + 6	0.5^1 (6) + 6
2	0.5 (0.5 (6) + 6) + 6	0.5^2 (6) + 0.5 (6) + 6
3	0.5 (0.5 (0.5 (6) + 6) + 6) + 6	$0.5^3(6) + 0.5^2(6) + 0.5(6) + 6$
4	0.5 (0.5 (0.5 (0.5 (6) + 6) + 6) + 6) + 6	$0.5^4(6) + 0.5^3(6) + 0.5^2(6) + .5(6) + 6$
5	0.5 (0.5 (0.5 (0.5 (0.5 (6) + 6) + 6) + 6) + 6) + 6	$0.5^5(6) + 0.5^4(6) + 0.5^3(6) + 0.5^2(6) + .5(6) + 6$

LM-145. a) Twice as much.

b) A geometric series is formed; $6 \Diamond \sum_{k=0}^{n} (0.5)^k$ c) $D \Diamond \sum_{k=0}^{n} (0.5)^k$

LM-146. a) 75% b) $D \Diamond \sum_{k=0}^{n} (0.75)^k$ c) The dosage is 3 mg.

LM-148. Max = $D \Diamond \lim_{n \to \infty} \sum_{k=0}^{n} (E)^k = \frac{D}{1\ E}$ LM-149. 75%

LM-150. $2.4 < D < 7.2$

LM-151. The prescription needs to be between 4.5 and 4.8 mgs.

LM-152. a) $1 + \ = \ ^2$, $1 + 2 = \ ^3$ LM-153. a) b)

LM-154. b) area under curve from t = 0 to t = 2

c) divided into 20 rectangles of width 0.1

d) $\boxed{\sum_{n=1}^{20} 0.6(0.1n)^2}$

e) $0.5[6(0)^2 + 12(0.1)^2 + 12(0.2)^2 + ... + 12(1.9)^2 + 6(2)^2]$
f) Right endpoint rectangles give 17.22 mi, left=14.82, trap=16.02

LM-155. $y = -2x + 10$ LM-156. $2\sec^2$

LM-157. a) -6 b) -5 c) -4.5 d) -4.1
e) It gets closer to -4.

LM-158. a) 97 b) $e^2 - 3$ c) $\frac{e^4}{3}$ d) $\boxed{\frac{4}{e^2 \pm 1}}$

LM-159. a) -5 b) 4.667 LM-160. 17.06 hours LM-161. $k = \ln m$

LM-163. a) $y = 14.715 \cdot e^{-0.16x}$ b) 14.715 mg
c) about 5.2 mg/cm^3 d) 9.94 hours

LM-164. a) $y = 0.28 \cdot e^{0.14x}$
c) It is too big at about 4 days, 3 hours, and 24 minutes.

LM-165. \$512.67 daily and \$512.71 continuously LM-166. b) $\boxed{\frac{1}{x \ a}}$ c) sin x

LM-167. a) possible b) not possible
c) possible—example: f(x) = sin (x) d) not possible

LM-168. By comparing the dominant terms of p(x) and q(x).

LM-169. a) $\boxed{\frac{8}{13}}$, $\boxed{\frac{13}{21}}$, $\boxed{\frac{21}{34}}$ b) $\boxed{1}$ LM-170. a) $\boxed{7 \pm 4\sqrt{3}}$ b) $\boxed{\frac{15+5\sqrt{5}+3\sqrt{2} \pm \sqrt{10}}{23}}$

LM-171. a) 64, 9, 4, 3, $1 + \sqrt{3}$ b) + 1

LM-172. 90'LM-173. b LM-174. $\boxed{x(3 + \sqrt{9 \pm x^2})}$

LM-175. $f(x) = \frac{2(x+7)(x \pm 2)}{(x+5)(x \pm 2)}$ LM-176. 12, 4, $\frac{4}{3}$

LM-177. $x = \frac{\mathbf{180 \pm A}}{\mathbf{2}}$ LM-178. $\frac{a}{(\ 2)^{n\ 1}}$

LM-179.

n	angle a_n	angle measure t(n)	error, e(n) 60 – t(n)	angle measure
1	a_1	20	40	A
2	a_2	80	-20	$\frac{180 \pm A}{2}$
	etc.	etc.		

LM-180. a) If you are at the limit then every fold from that point on will be the limit value. b) $L = \frac{180 \pm L}{2}$ c) $L = 60^\circ$

LM-181. In the first step, where she assumed the limit exists.

LM-182. a)

n	angle a_n	(n) special case	(n) general case
1	a_1	60 – 40	60 +
2		60 + 20	60 – $\frac{}{2}$
3		60 – 10	60 + $\frac{}{4}$
4		60 + 5	60 – $\frac{}{8}$
5		60 – 2.5	60 + $\frac{}{16}$
6		60 + 1.25	60 – $\frac{}{32}$

b) $60 + \frac{(\ \ 1)^{n \pm 1} \lozenge}{2^{n \pm 1}}$ c) The numerator of the fraction only changes sign, while the denominator approaches • .

LM-184. b) 20 seconds c) a $0.131\ \text{ft/s}^2$; speed = $78\frac{2}{3}$ ft/s

LM-185. a) 4055 b) 5 days from his measurement.

LM-186. a(x): Yes, $y = \frac{1}{2}$. b(x): No, because b(x) approaches infinity.
c(x): Yes, because y=1.

LM-187. $25(2x)(y)^{24} = 50xy^{24}$

LM-188. a) $\ln 6$ b) $e^3 - 4$ c) $\boxed{\frac{\ln 28}{3}}$ d) e^5

LM-189. a) b) no limit c) 1 d)

LM-190. a) 27 b) 6.75 c) 23.5 d) unknow

LM-191. $\frac{8x^9}{y^{12}}$ LM-192. $2xh + h + h^2$

LM-193. a) $\boxed{\frac{4}{11}}$ b) $\boxed{\frac{75}{11}}$ c) $\boxed{\frac{25}{37}}$ LM-194. b) $y = -\frac{3}{4}x + \frac{25}{4}$

LM-195. a) decreasing b) increasing c) increasing

LM-196. 42.86 mph LM-197. $\boxed{\frac{761}{280} \quad 2.718}$ LM-199. infinity

LM-200. a) 2.929 b) 3.598 c) 5.187 d) 6.793

LM-203. First is the largest, last is the smallest.

$\frac{2}{n}$ $\frac{1}{n^2}$ $\frac{1}{2n^2 \quad n}$

LM-204. • ; if not, the harmonic series would have a finite sum.

LM-205. b) $\boxed{(1 \pm \frac{1}{2}) + (\frac{1}{3} \pm \frac{1}{4}) + \ldots}$ c) $\boxed{1 \pm (\frac{1}{2} \pm \frac{1}{3}) \pm (\frac{1}{4} \pm \frac{1}{5}) \pm \ldots}$ d) The exact sum is $\ln 2 = 0.693$.

LM-206. x = 2.4, y = LM-207. b) 63.72 ft

LM-208. a) $\frac{2x(x \ 1)}{3y}$ b) $2x + h$ LM-209. x = 4.5

LM-210. a) $x \ \pi \ 3$ b) $x = 3$ c) $y = -5$

LM-211. b) $S = 2x^2 + \frac{90}{x}$ LM-212. a) 74.1° b) $\sqrt{53}$ LM-213. $(-3, 3\sqrt{3})$

